STUDENT NAME: ______________________

TEACHER: _____________________________

HURLSTONE
AGRICULTURAL
HIGH
SCHOOL

2018

HIGHER
SCHOOL
CERTIFICATE
Task 4 Assessment - Trial

Mathematics

Examiners	Ms L Yuen, Mr. S Faulds, Ms. M Sabah, Mr. D. Potaczala, Ms T Tarannum, Mr. R. Raswon

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA-approved calculators may be used
- A Reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 19)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

SECTION I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10

Question 1

What is 0.0000205 written in scientific notation, correct to 2 significant figures?

A. 2.1×10^{5}

B. 2.05×10^{5}

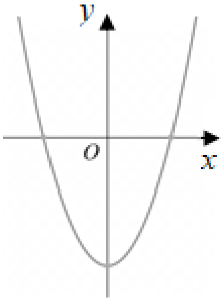
C. 2.05×10^{-5}

D. 2.1×10^{-5}

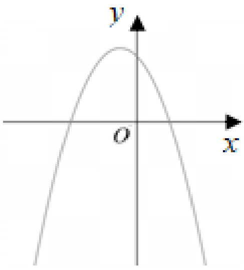
Question 2

Which diagram best shows the graph of the parabola y = $-(x+2)(x-1)$?

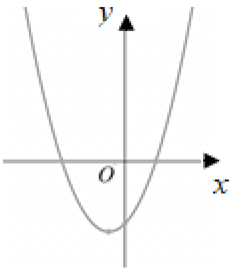
A.



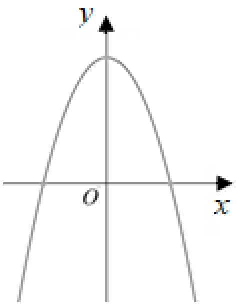
B.



C.

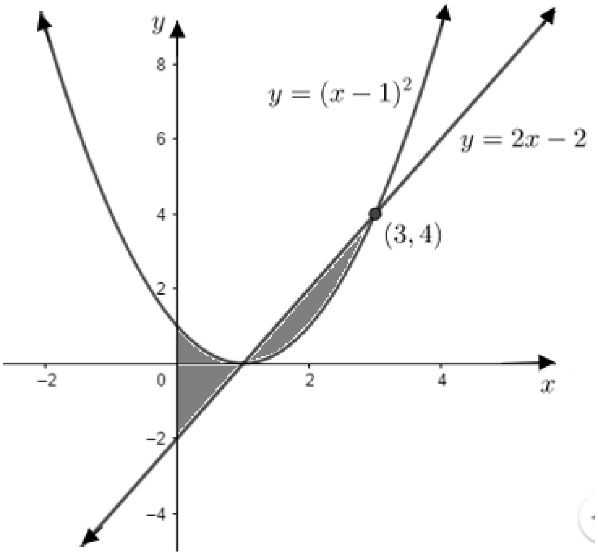


D.



Question 3

The diagram shows the parabola $y = (x - 1)^2$ and the line $y = 2x - 2$ intersecting at (1, 0) and (3, 4). Which of the following expression gives the area of the shaded regions bounded by the parabola, the line and the y-axis?



A. $\int_0^3 2x-2-(x-1)^2\,dx$

B. $\int_1^3 (x-1)^2-2x-2dx+\int_0^1 2x-2-(x-1)^2\,dx$

C. $\left|\int_0^3 2x-2-(x-1)^2\,dx\right|$

D. $\int_0^1 \left[(x-1)^2-2x+2\right]dx+\int_1^3 \left(2x-2\right)-(x-1)^2\,dx$

Question 4

What is the value of the derivative of the function $y = 2^x$ when $x = 1$?

A. $2\ln 2$

B. $\ln 2$

C. 1

D. 0

Question 5

What is the exact value of $\sec 30^o + \tan 30^o$?

A. $\frac{5\sqrt{3}}{6}$

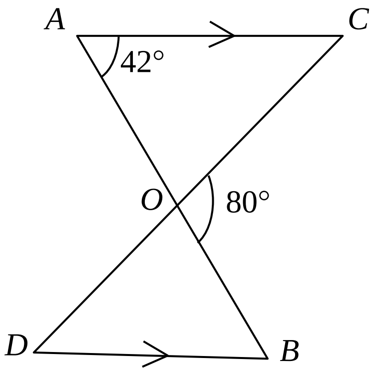
B. $\frac{3\sqrt{3}}{2}$

C. $\frac{5\sqrt{3}}{3}$

D. $\sqrt{3}$

Question 6

In the figure, straight lines *AB* and *CD* intersect at a point *O*. *AC* || *DB*.
What is the size of $\angle BDO$?



A. 32°

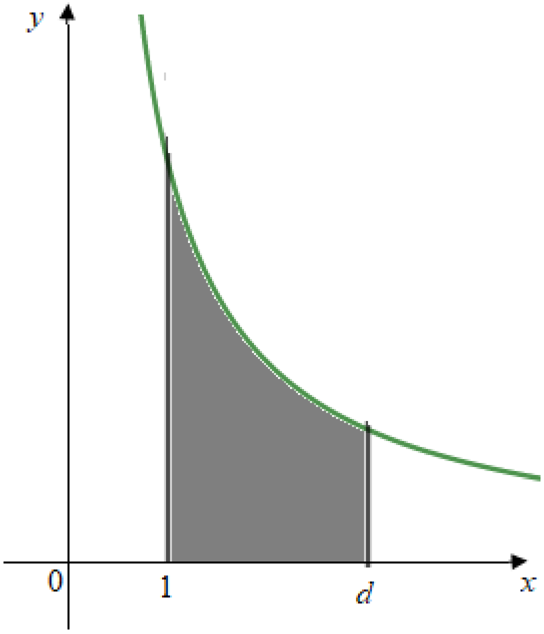
B. 38°

C. 42°

D. 48°

Question 7

The diagram shows the area under the curve $y = \frac{2}{x}$ from $x = 1$ to $x = d$.



What value of d makes the shaded area equal to 2?

A. e

B. $e + 1$

C. $2e$

D. e^2

Question 8

A bag contains 12 red marbles and x green marbles. If one marble is taken out at random, the probability of drawing a green marble is $\frac{4}{7}$. What is the value of x?

A. 4

B. 12

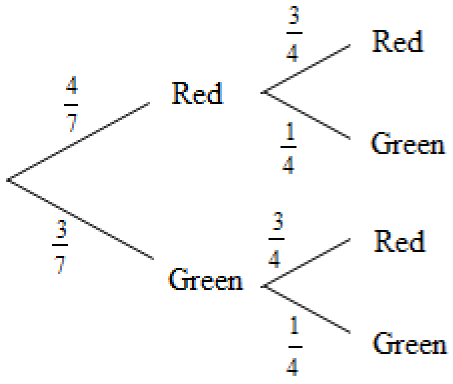
C. 16

D. 20

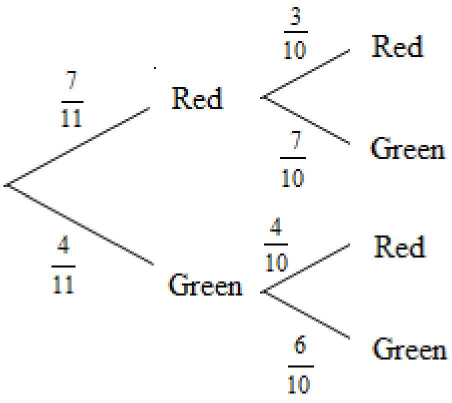
Question 9

John has 2 bags of colour blocks. Bag *A* contains 4 red blocks and 3 green blocks.
Bag *B* contains 3 red blocks and 1 green block. John choose a block from one of the bags.
Which tree diagram could be used to determine the probability that John choose a red block?

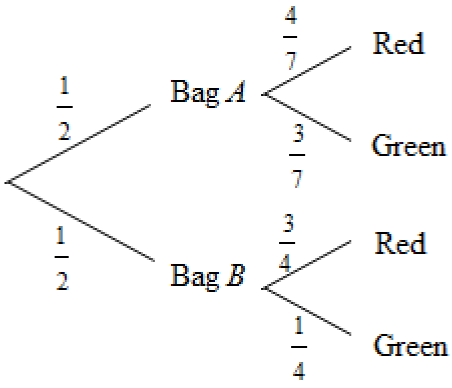
A.



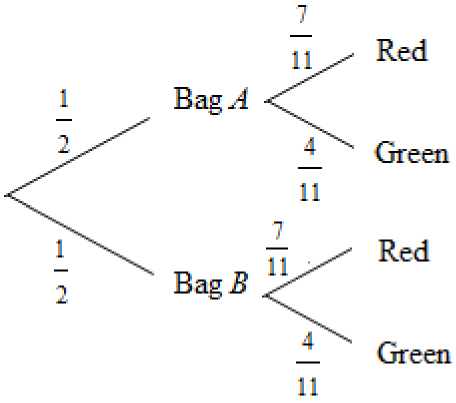
B.



C.



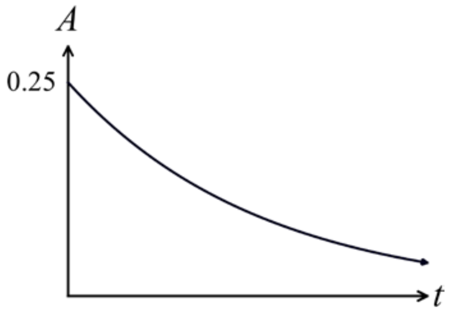
D.



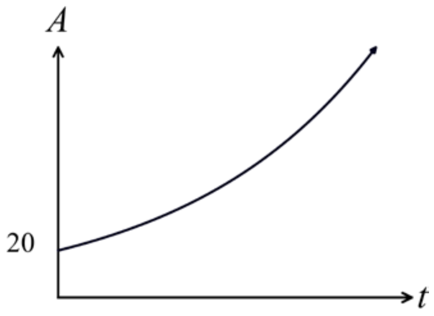
Question 10

The amount of a substance (A) is initially 20 units. The rate of change in the amount is given by $\frac{dA}{dt} = 0 \cdot 25A$. Which graph shows the amount of the substance over time?

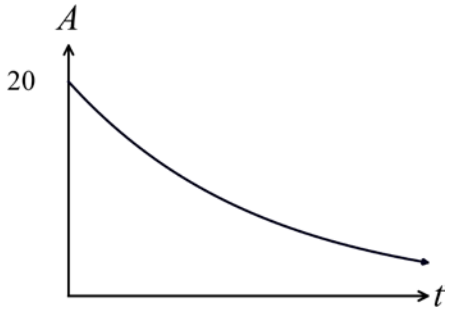
A.



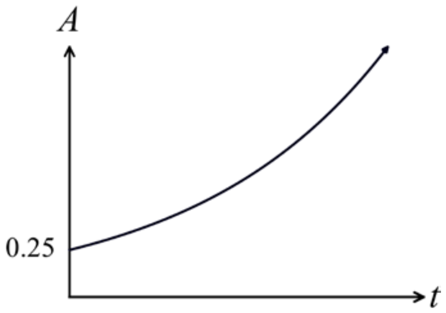
B.



C.



D.



SECTION II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section.

Answer each question in a **SEPARATE** writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new booklet

(a) Simplify $2x-(13-x)+3$. **1**

(b) Solve $|2x-7|=8$. **2**

(c) Factorise $2x^2+11x-21$. **2**

(d) Rationalise the denominator of $\frac{\sqrt{7}}{1-\sqrt{2}}$. **2**

(e) The equation of a parabola is $x^2=16(y-2)$.

- (i) Find the coordinates of the focus of the parabola. **2**
- (ii) Sketch the parabola and indicate the coordinates of the vertex, focus and directrix. **2**

(f) (i) Find the domain and range of the function $f(x)=\sqrt{36-x^2}$. **2**

(ii) On a number plane, shade the region that satisfies the inequality **2**

$$y<\sqrt{36-x^2}.$$

End of Question 11

Question 12 (15 marks) Start a new booklet

(a) Differentiate $(3x+5)\ln x$ **2**

(b) Differentiate $\dfrac{e^{3x}}{2x+1}$ **2**

(c) Find $\int (x^{\frac{3}{2}} + x^{-2})\ dx$ **2**

(d) The gradient at any point (x, y) of a curve is given by

$\dfrac{dy}{dx} = 3x^2 - 2x + k$. If the curve touches the x-axis at the point (2, 0), find

(i) the value of k. **2**

(ii) the equation of the curve. **2**

(e) Consider the curve $y = 2x^3 - 3x^2$.

(i) Find the stationary points and determine their nature. **3**

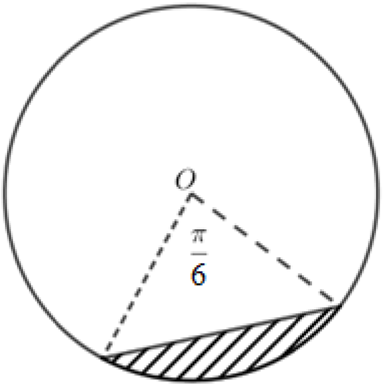
(ii) Find the point of inflexion for the curve. **1**

(iii) Sketch the curve labelling the stationary points, point of inflexion and x-intercepts. **1**

End of Question 12

Question 13 (15 marks) Start a new booklet

(a) The area of the minor segment of the circle pictured below is $100\ \text{m}^2$. **2**
Find the radius of the circle to the nearest metre.



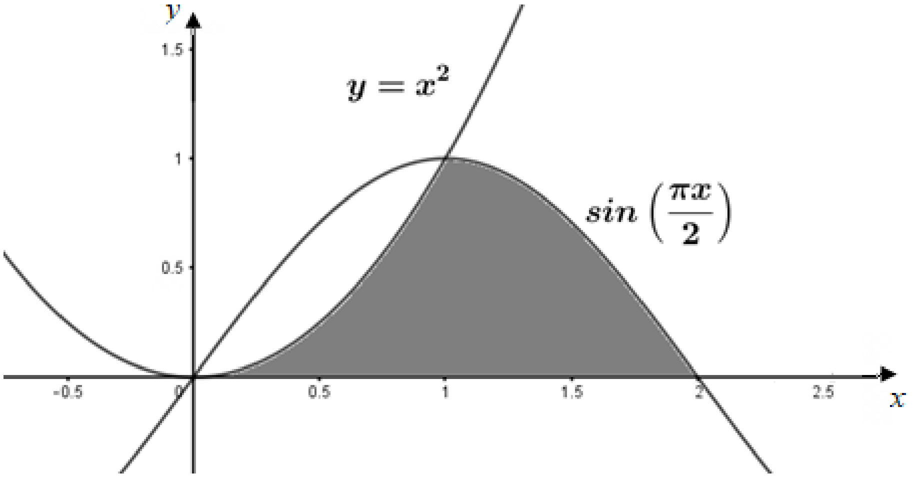
(b) (i) Show that $\frac{d}{dx}\log_e\left(\tan^2 x\right)=\frac{2}{\sin x\cos x}$ **2**

(ii) Hence find in simplest exact form the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\cos x\sin x}dx$ **2**

(c) Sketch the graph of $y=-4\sin(3x)$ for $0\leq x\leq 2\pi$ showing all x-intercepts. **2**

Question 13 continues on page 12

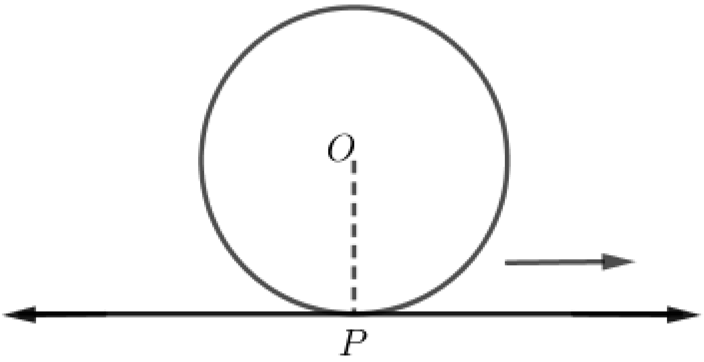
(d) The shaded region in the diagram is bounded by the curves $y = \sin\frac{\pi x}{2}$, $y = x^2$ and the x-axis.



(i) Given that the two curves intersect at $x = 1$, calculate the exact area of the shaded region. **2**

(ii) Write an expression that will give the volume of the solid of revolution that is formed when the shaded region is rotated about the x-axis. **Do NOT evaluate your expression.** **1**

(e) A train wheel of radius 40 cm and centre *O* rolls along a horizontal track as shown in the diagram below. *P* is a point on the wheel where the wheel touches the track before it starts to roll.



(i) Through what angle does *P* rotate about *O* in radians after the wheel rolls 1 metre? **1**

(ii) What would be the vertical height of *P* above he track after the wheel rolls 1 metre? **3**

End of Question 13

Question 14 (15 marks) Start a new booklet

(a) (i) Without using calculus, sketch the graph of $y = \log_e x$. **1**

(ii) On the same sketch, find, graphically, the number of solutions of the equation **2**

$$\log_e x - x = -2$$

(b) (i) For the function $f(x) = xe^{-2x} + 1$, ***show*** that the first derivative is $f'(x) = e^{-2x} - 2xe^{-2x}$. **2**

(ii) Find the values of *x* for which $f(x)$ is increasing. **2**

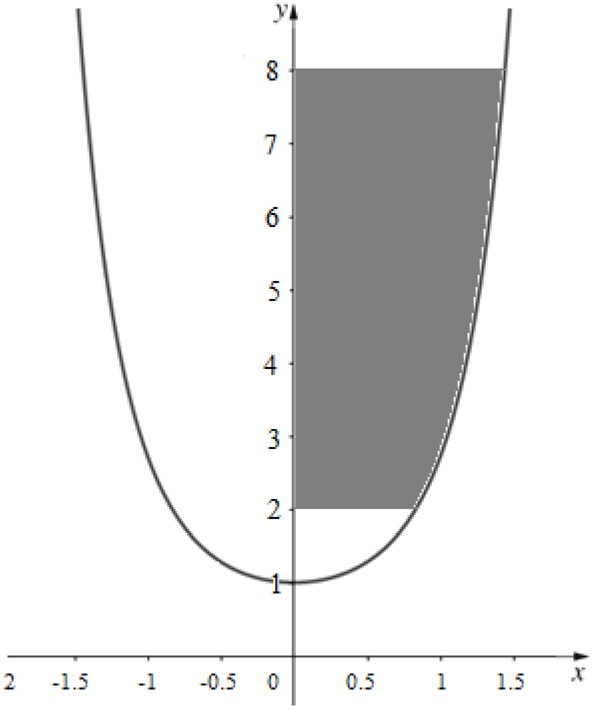
(c) (i) Find the primitive function, $F(x)$, for: **3**

$$f(x) = 3e^{x-1} + \frac{x}{3x^2 - 2}\ , \text{ if } F(1) = -3.$$

(ii) Hence, find F(3), correct to two decimal places. **1**

Question 14 continues on page 14

(d) The diagram below shows a shaded area enclosed between the y-axis, the lines $y = 2$ and $y = 8$, and the curve $y = e^{x^2}$.



(i) Write $y = e^{x^2}$ in logarithmic form. **1**

(ii) The area shown above is rotated about the y-axis. Show that the volume created is given by: **1**

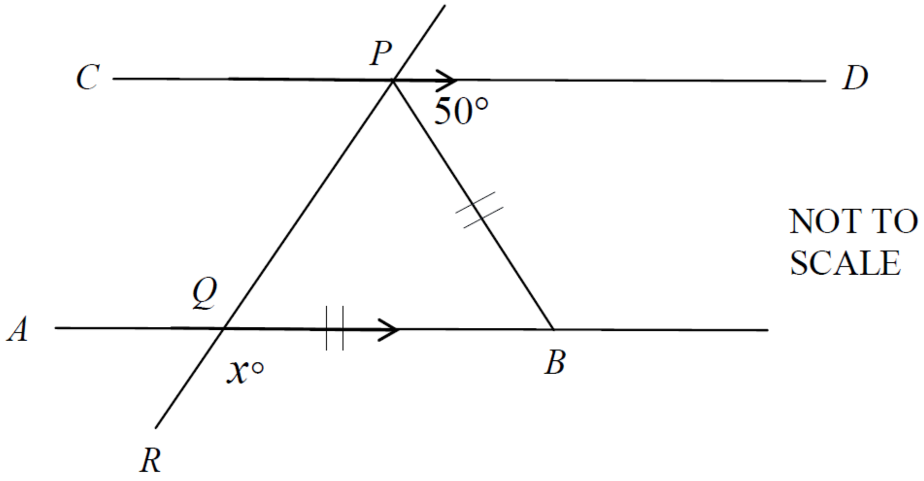
$$V = \pi \int_2^8 \log_e y \, dy$$

(iii) Use one application of Simpson's Rule to approximate the volume, V, correct to two decimal places. **2**

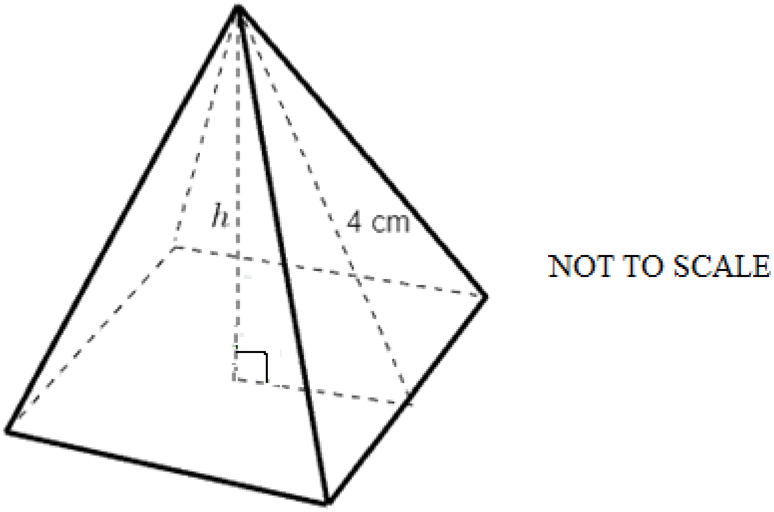
End of Question 14

Question 15 (15 marks) Start a new booklet

(a) In the diagram, CD is parallel to AB, $PB = QB$, $\angle BPD = 50°$ and $\angle BQR = x°$. Find the value of x, giving complete reasons. **3**



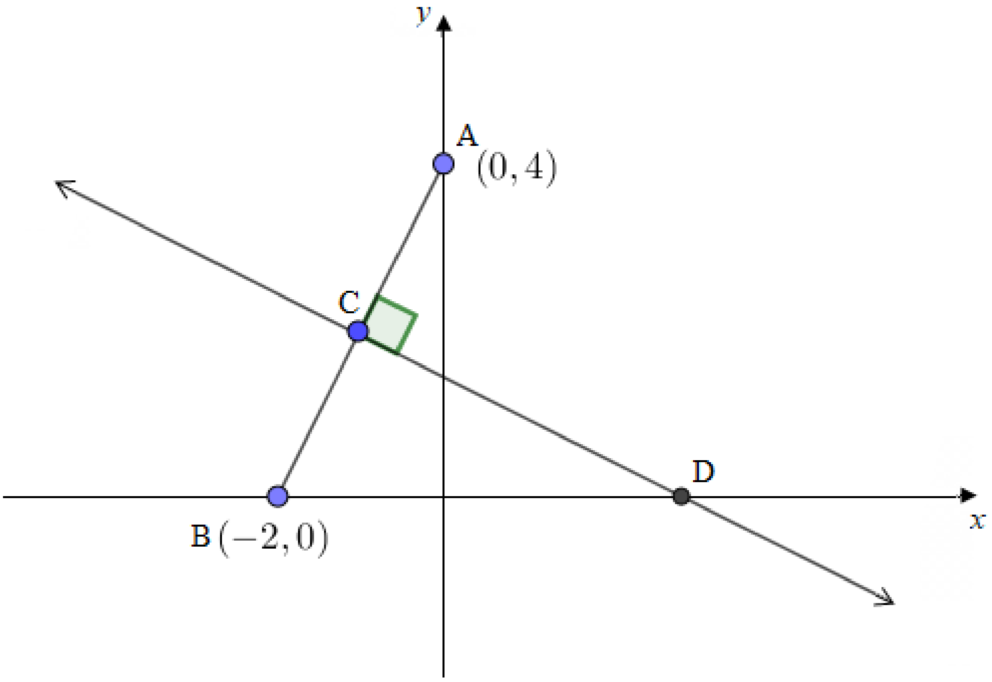
(b) A diamond is to be cut in the shape of a square pyramid, with a slant height 4 cm and a perpendicular height h as shown in the diagram below.



Show that the volume of the diamond can be expressed as $V = \frac{4h}{3}\left(16 - h^2\right)$. **2**

Question 15 continues on page 16

(c) The diagram shows the points $A(0,\ 4)$ and $B(-2,\ 0)$. C is the midpoint of AB. Line CD is drawn perpendicular to AB and crosses the x-axis at D. Find the equation of line CD in general form. **3**

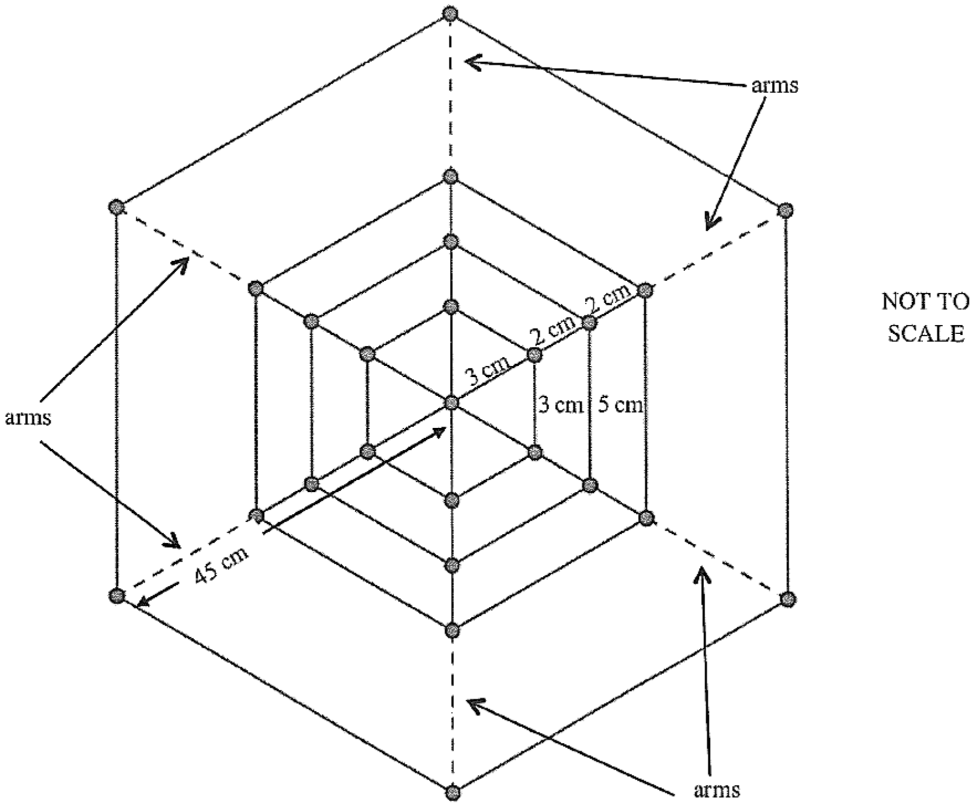


(d) A person invests $800 at the beginning of each year in a superannuation fund. Compound interest is paid at 10% per annum on the investment. **2**
The first $800 was invested at the beginning of 2016 and the last is to be invested at the beginning of 2045.

Calculate to the nearest dollar, the amount to which the total investment will have grown by the beginning of 2046.

Question 15 continues on page 17

(e) Incey Wincey spider makes a web in the shape of concentric regular hexagons. First he makes the 6 arms which are 45 cm long. He then makes the sides of each regular hexagon. The first is 3 cm from the centre along each arm. Each successive regular hexagon is 2 cm further along the arm. The last hexagon is at the end of the arms.



(i) How many regular hexagons does Incey Wincey create? **2**

(ii) Find the total length of the web, including the arms, created by Incey Wincey. **3**

End of Question 15

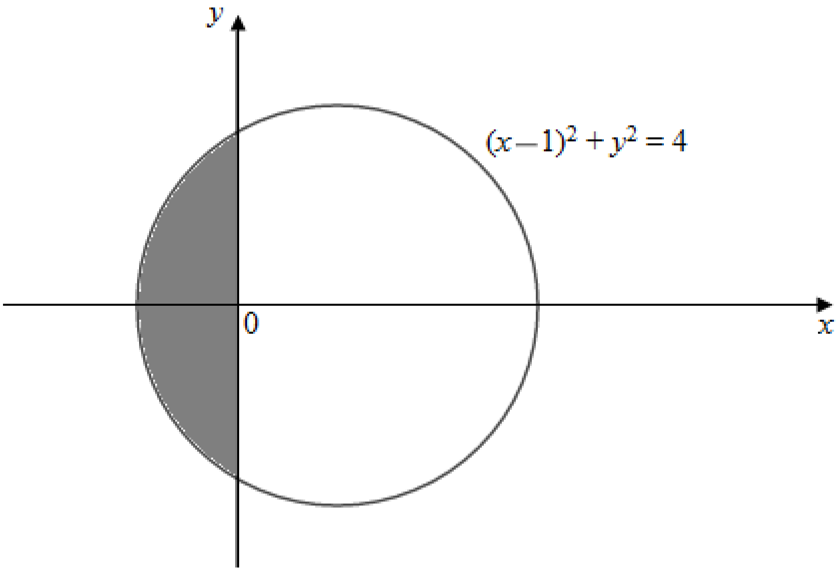
Question 16 (15 marks) Start a new booklet

(a) The rate of change of the temperature, T, of a kettle, which initially is at 100°C, is given by **3**

$$\frac{dT}{dt} = \frac{t}{20} - k$$

where t is the time in minutes and k is a constant.
Twenty minutes after the kettle had boiled, the water temperature had fallen to 30°C. Find T in terms of t.

(b) The diagram shows the circle $(x - 1)^2 + y^2 = 4$. Find the volume of the solid of revolution if the shaded region is rotated about the x-axis. **3**



Question 16 continues on page 19

(c) An underground storage tank is in the shape of a rectangular prism with a floor area of 12 m^2 and a ceiling height of 2 m. At 2 p.m. one Sunday, rain water begins to enter the storage tank. The rate at which the volume V of the water changes over time t hours is given by

$$\frac{dV}{dt} = \frac{24t}{t^2+15}$$

where $t = 0$ represents 2 p.m. on Sunday, and where V is measured in cubic metres. The storage tank is initially empty.

(i) Show that the volume of water in the tank at time t is given by **3**

$$V = 12\ln\left(\frac{t^2+15}{15}\right), t \geq 0$$

(ii) Find the time when the tank will be completely filled with water if the water continues to enter the tank at the given rate. Express your answer to the nearest minute. **3**

(iii) The owners return to the house and manage to simultaneously stop the water entering the tank and start the pump in the tank. This occurs at 6 p.m. on Sunday. The rate at which the water is pumped out of the tank is given by **3**

$$\frac{dV}{dt} = \frac{t^2}{k} \text{ where k is a constant}$$

At exactly 8 p.m. the tank is emptied of water. Find the value of k. Express your answer correct to 4 significant figures.

End of Question 16

End of Paper

2018 MC answer for Trial Mathematics

1.D 2.B 3.D 4.A `5.D

6.B 7.A 8.C 9.C 10.B

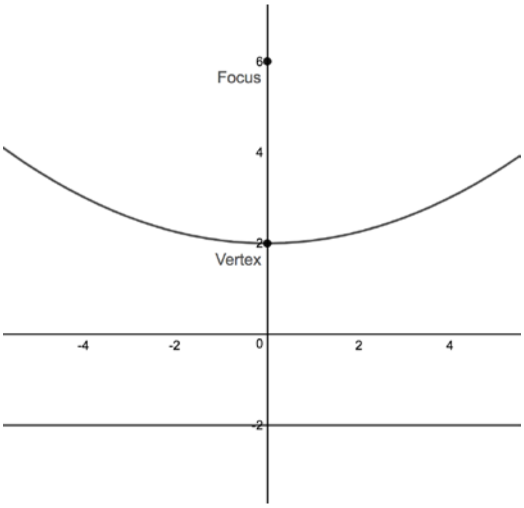
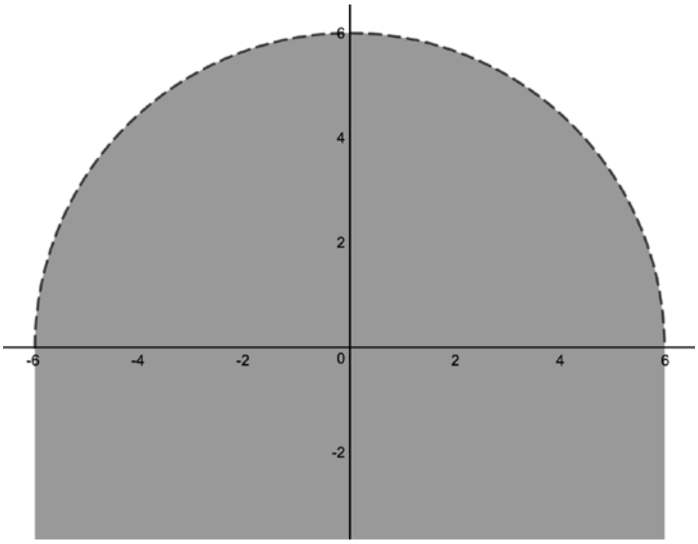
Year 12 2018 Mathematics Task 4 Trial

Question No. 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P3 - performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
P4 - chooses and applies appropriate arithmetic, algebraic, graphical and geometric techniques
P5 - understands the concept of a function and the relationship between a function and its graph
H9 - communicates using mathematical language, notation, diagrams and graphs

Outcome	**Solutions**	**Marking Guidelines**
P3 P4 H9	a) $\begin{aligned} &2x-(13-x)+3 \\ &=2x-13+x+3 \\ &=3x-10 \end{aligned}$	**1 mark** Correct answer
P3 P4 H9	b) $\begin{aligned} 2x-7&=8 \\ 2x&=15 \\ x&=7.5 \end{aligned}$ $\qquad \begin{aligned} -(2x-7)&=8 \\ 2x&=1 \\ x&=-\frac{1}{2} \end{aligned}$	**2 marks** Correct answer **1 mark** Substantial working
P3 P4 H9	c) $(2x-3)(x+7)$	**2 marks** Correct answer **1 mark** One error
P3 P4 H9	d) $\begin{aligned} \frac{\sqrt{7}}{1-\sqrt{2}} &= \frac{\sqrt{7}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} \\ &= \frac{\sqrt{7}+\sqrt{14}}{1-\sqrt{2}+\sqrt{2}-2} \\ &= -\left(\sqrt{7}+\sqrt{14}\right) \end{aligned}$	**2 marks** Correct answer **1 mark** Substantial working

P4 P5 H9	e) i. Vertex $\rightarrow (0,2)$ $a=4$ $\therefore$Focus $\rightarrow (0,6)$	**2 marks** Correct answer **1 mark** One error
	ii. 	**2 marks** Correct answer **1 mark** Missing focus or directrix or one error
P4 P5 H9	f) i. Domain: $-6 \le x \le 6$ Range: $0 \le y \le 6$	**2 marks** Correct answer **1 mark** One error
	ii. 	**2 marks** Correct answer **1 mark** One error

Question No. 12 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P6 relates the derivative of a function to the slope of its graph
P7 determines the derivative of a function through routine application of the rules of differentiation
H5 Applies appropriate techniques from the study of differential and integral calculus to solve problems
H6 uses the derivative to determine the features of the graph of a function

Outcome	**Solutions**	**Marking Guidelines**
P7	**a)** $\frac{dy}{dx}=(3x+5)\frac{1}{x}+\ln x(3)$ $=3+\frac{5}{x}+3\ln x$	Award 2 for correct solution Award 1 for substantial progress towards solution
	b) $\frac{dy}{dx}=\frac{(2x+1)e^{3x}(3)-e^{3x}(2)}{(2x+1)^2}$ $=\frac{e^{3x}(6x+3-2)}{(2x+1)^2}$ $=\frac{e^{3x}(6x+1)}{(2x+1)^2}$	Award 2 for correct solution Award 1 for substantial progress towards solution
	c) $y=\frac{2x^{\frac{5}{2}}}{5}+\frac{x^{-1}}{-1}+\mathrm{C}$ $=\frac{2x^{\frac{5}{2}}}{5}-\frac{1}{x}+\mathrm{C}$	Award 2 for correct solution Award 1 for substantial progress towards solution
P6	**d)(i)** The curve touches the x-axis at the point (2,0), Hence the gradient of the tangent at (2,0) is 0. $\therefore \frac{dy}{dx}=0,\ at\ \ x=2$ $3x^2-2x+k=0$ $3(2)^2-2(2)+k=0$ $12-4+k=0$ $\therefore k=-8$	Award 2 for correct solution Award 1 for substantial progress towards solution
H5	**(ii)** $y=\int(3x^2-2x-8)dx$ $=x^3-x^2-8x+\mathrm{C}$ It passes through (2,0) $\therefore\ \ 2^3-2^2-8(2)+\mathrm{C}=0$ $-12+\mathrm{C}=0$ $\mathrm{C}=12$ The equation of the curve is $y=x^3-x^2-8x+12$	Award 2 for correct solution Award 1 for substantial progress towards solution

H6

(e)(i)

$$y = 2x^3 - 3x^2$$

$$\frac{dy}{dx} = 6x^2 - 6x$$

$$\text{put } \frac{dy}{dx} = 0$$

$$6x^2 - 6x = 0$$

$$6x(x-1) = 0$$

$$\therefore x = 0 \quad or \quad x = 1$$

$$y = 0 \qquad y = -1$$

x	−1	-0.5	0	0.5	1	1.5	2
$\frac{dy}{dx}$	12	4.5	0	–1.5	0	4.5	12

/ --- \ ---- /

Hence (0,0) is a local max point and (1,– 1) is a local min. point.

Or

$$\frac{d^2y}{dx^2} = 12x - 6$$

at $x = 0,\ y = 0$ at $x = 1,\ y = -1$

$$\frac{d^2y}{dx^2} = -6 \qquad \frac{d^2y}{dx^2} = 6$$

$$< 0 \qquad > 0$$

Hence (0,0) is a local max point and (1,– 1) is a local min. point.

Award 3 for correct solution

Award 2 for substantial progress towards solution

Award 1 for limited progress towards solution

(ii)

For the point of inflexion

$$\text{put } \frac{d^2y}{dx^2} = 0$$

$$12x - 6 = 0$$

$$x = \frac{1}{2}$$

$$y = 2\times\left(\frac{1}{2}\right)^3 - 3\times\left(\frac{1}{2}\right)^2$$

$$= -\frac{1}{2}$$

Testing concavity

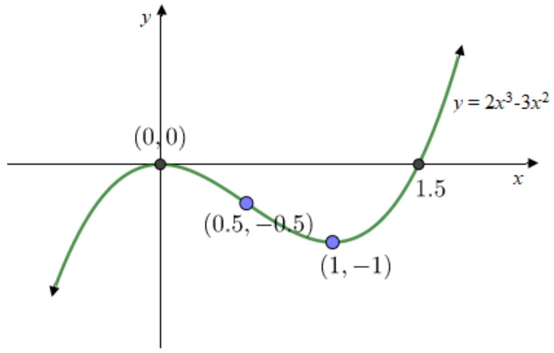
x	0	0.5	1
$\frac{d^2y}{dx^2}$	–6	0	6

\ ---- /

Since concavity changes, there is a point of inflexion at $\left(\frac{1}{2}, \frac{-1}{2}\right)$

Award 1 for correct solution

(iii)



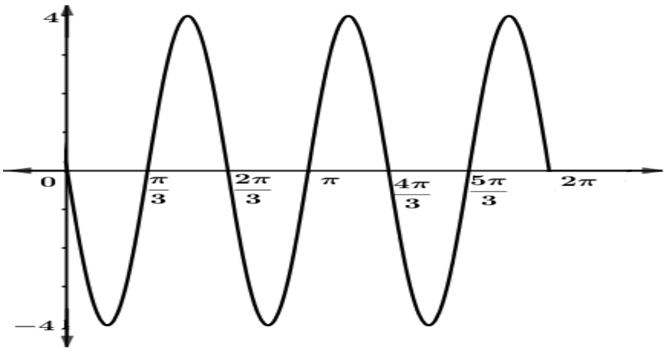
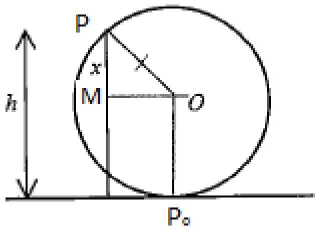
Award 1 for correct graph

Question No. 13 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 *applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve*
H8 *uses techniques of integration to calculate areas and volumes*
H9 *communicates using mathematical language, notation, diagrams and graphs*
P3 *performs routine arithmetic and algebraic manipulation involving surds and simple rational expressions.*
P4 *chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques*

Outcome	Solutions	Marking Guidelines
H5, P3	**a)** $100=\frac{1}{2}r^2\left(\frac{\pi}{6}-\sin\left(\frac{\pi}{6}\right)\right)$ $200=r^2\left(\frac{\pi}{6}-\frac{1}{2}\right)$ $200=r^2\left(\frac{\pi-3}{6}\right)$ $r^2=\frac{1200}{\pi-3}$ $r=\sqrt{\frac{1200}{\pi-3}}\approx 92\cdot 05984992\approx 92m$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution
H5,P4	**b) (i)** $\frac{d}{dx}\log_e\left(\tan^2 x\right)$ $=\frac{2\tan x\sec^2 x}{\tan^2 x}=\frac{2\sec^2 x}{\tan x}$ $=\frac{2\cos x}{\sin x\cos^2 x}=\frac{2}{\sin x\cos x}=RHS$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution
H5,P4	**(ii)** $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\cos x\sin x}dx=\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{2}{\cos x\sin x}dx$ $=\frac{1}{2}\left[\log_e\tan^2 x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\frac{1}{2}\left[\log_e\tan^2\left(\frac{\pi}{3}\right)-\log_e\tan^2\left(\frac{\pi}{4}\right)\right]$ $=\frac{1}{2}\left[\log_e\left(\sqrt{3}\right)^2-\log_e\left(1\right)\right]=\frac{1}{2}\ln\left(3\right)$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution

H9	**c)**  	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution
H8	**d) (i)** $A=\int_0^1 x^2dx+\int_1^2 \sin\left(\frac{\pi x}{2}\right)dx$ $=\left[\frac{x^3}{3}\right]_0^1+\left[\frac{-2}{\pi}\cos\left(\frac{\pi}{2}x\right)\right]_1^2$ $=\left(\frac{1}{3}-0\right)-\left(\frac{2}{\pi}\right)\left(\cos(\pi)-\cos\left(\frac{\pi}{2}\right)\right)=\frac{1}{3}-\frac{2}{\pi}(-1)=\frac{1}{3}+\frac{2}{\pi}units^2$	**Award 2** marks for the correct answer. **Award 1** mark for substantial progress towards the solution
H8	**(ii)** $V=\pi\int y^2dx$ $V=\pi\left[\int_0^1 x^4dx+\int_1^2 \sin^2\left(\frac{\pi x}{2}\right)dx\right]$	**Award 1** mark for the correct answer.
H5	**e) (i)** $l=r\theta$ $100=40\theta$ $\theta=\frac{5}{2}radians$ **(ii)**  Let *h* be the height of *P* above the track after the wheel has rolled 1 metre. The angle required for the triangle: $\angle POM=\frac{5}{2}-\frac{\pi}{2}radians$ $\sin\left(\frac{5}{2}-\frac{\pi}{2}\right)=\frac{x}{40}$ $x=40\sin\left(\frac{5-\pi}{2}\right)$ $x\approx 32$ So the height of *P* above the track is 32+40 = 72 cm	**Award 1** mark for the correct answer. **Award 3** marks for the correct answer. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.

Year 12 Mathematics Trial HSC Examination 2018

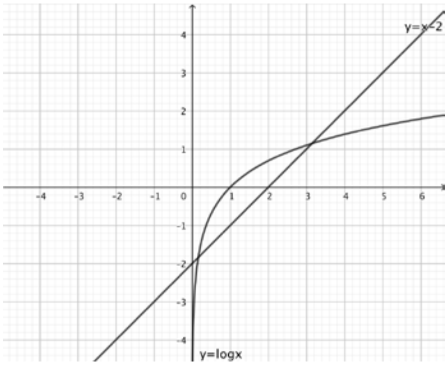
Question No. 14 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 **applies appropriate techniques from the study of calculus, geometry,** probability, trigonometry and series **to solve problems**

H6 **uses the derivative to determine the features of the graph of a function**

H8 **uses techniques of integration to calculate areas and volumes**

Outcome	Solutions	Marking Guidelines
H5	**(a)(i)**  	**1 mark** Correct graph of the log function.
H5	(ii) The equation $\log_e x - x = -2$ can be written as $\log_e x = x - 2$. Draw the line y = x – 2 on the same graph (see above). There are two points of intersection, hence there must be **two solutions** to the equation.	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.
H5	**(b)(i)** $f(x) = xe^{-2x} + 1$ $f'(x) \quad = uv' + vu' + \frac{d}{dx}1 \qquad$ where $\quad u = x \qquad v = e^{-2x}$ $= -2xe^{-2x} + e^{-2x} \qquad u' = 1 \qquad v' = -2e^{-2x}$ $= e^{-2x} - 2xe^{-2x}$ as required	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.
H6	**(ii) The function is increasing when:** $e^{-2x} - 2xe^{-2x} > 0$ $e^{-2x}(1-2x) > 0$ $1 - 2x > 0\left(\text{since } e^{-2x} > 0 \text{ always}\right)$ $x < \frac{1}{2}$	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.
H5	**(c) (i)** $f(x) = 3e^{x-1} + \frac{x}{3x^2-2}$ $F(x) = 3e^{x-1} + \frac{1}{6}\ln\left(3x^2-2\right) + c$ since $F(1) = -3$ $F(1) = 3e^0 + \frac{1}{6}\ln(3\times1-2) + c$ $-3 = 3 + 0 + c$ $c = -6$ $\therefore$ The primitive function is $F(x) = 3e^{x-1} + \frac{1}{6}\ln\left(3x^2-2\right) - 6$	**3 marks** Correct solution **2 marks** Single error in finding primitive or calculating constant of integration. **1 mark** Some progress towards a correct solution.

<table>
<tr>
<td>H5</td>
<td>(ii)
$$\begin{aligned}F\left(3\right)&=3e^{3-1}+\frac{1}{6}\ln\left(3\times3^2-2\right)-6\\&=16.70\quad\left(\text{correct to 2 dec. pl.}\right)\end{aligned}$$</td>
<td>1 mark
Correct answer.</td>
</tr>
<tr>
<td>H5</td>
<td>(d)(i) $y=e^{x^2}\Leftrightarrow x^2=\log_e y$</td>
<td>1 mark
Correct answer.</td>
</tr>
<tr>
<td>H8</td>
<td>(ii)
$$\begin{aligned}V&=\pi\int_a^b x^2dy\\&=\pi\int_2^8\log_e ydy\end{aligned}$$</td>
<td>1 mark
Correct solution.</td>
</tr>
<tr>
<td>H8</td>
<td>(iii)
<table>
<tr><td>y</td><td>2</td><td>5</td><td>8</td></tr>
<tr><td>$\log_e y$</td><td>0.693</td><td>1.609</td><td>2.079</td></tr>
</table>
Using Simpson’s Rule
$$\begin{aligned}\int_2^8\log_e ydy&\approx\frac{h}{3}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)\\&=\frac{3}{3}\left(0.693+4\times1.609+2.079\right)\\&=9.208\end{aligned}$$
Hence,
$$\begin{aligned}V&=9.208\times\pi\\&=28.93\text{units}^3\quad\text{(correct to 2 dec. pl.)}\end{aligned}$$</td>
<td>2 marks
Correct solution.
1 mark
Substantial progress towards correct solution.</td>
</tr>
</table>

Question No. 15 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 Applies appropriate techniques from the study of series, probability and geometry to solve problems
P4 Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

Outcome	**Solutions**	**Marking Guidelines**
P4 **H5**	**(a)** $\angle BQP = 180^\circ - x$ (angles on a straight line add to 180°) $\angle BPQ = \angle PQB$ (base angles of isosceles ΔBPQ) $= 180^\circ - x$ $\angle QBP = \angle BPD = 50^\circ$ (alternate angles in parallel lines are equal) $180^\circ - x + 180^\circ - x + 50 = 180^\circ$ (angle sum of triangle) $-2x = -230$ $\therefore \quad x = 115^\circ$	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution
	(b) Using Pythagoras Theorem, $x^2 + h^2 = 4^2$ $\therefore x = \sqrt{16 - h^2}$ So the lenth of the base is $2\sqrt{16 - h^2}$. Area of the base $= \left(2\sqrt{16 - h^2}\right)^2 = 4\left(16 - h^2\right)$ $\therefore$ Volume of the pyramid $V = \frac{1}{3} \times 4\left(16 - h^2\right) \times h$ $\therefore \quad V = \frac{4h}{3}\left(16 - h^2\right)$	2 marks : correct solution 1 marks : substantial progress towards correct solution
	(C) Midpoint $C = \left(\frac{0+2}{2}, \frac{4+0}{2}\right)$ $= \left(-1, 2\right)$ $m_{AB} = \frac{4}{2} = 2$ $\therefore m_{CE} = -\frac{1}{2}$ $y - 2 = -\frac{1}{2}\left(x + 1\right)$ $2y - 4 = -x - 1$ $\therefore \quad x + 2y - 3 = 0$	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution

P4	**(d)** $I_1 = 800(1.1)^{30}$ $I_2 = 800(1.1)^{29}$ . . $I_{30} = 800(1.1)^{1}$ Total investment $= 800(1.1)^{30} + 800(1.1)^{29} + \ldots + 800(1.1)^{1}$ $= 800\left\{(1.1)^{30} + (1.1)^{29} + \ldots + (1.1)^{1}\right\}$ GP with $a = 1.1,\ n = 30,\ r = 1.1$ $= 800\left(\frac{1.1\left(1.1^{30} - 1\right)}{1.1 - 1}\right)$ $= 800\left(\frac{1.1\left(1.1^{30} - 1\right)}{0.1}\right)$ $= 800\left(11\left(1.1^{30} - 1\right)\right)$ $= \$144\ 755$	2 marks : correct solution 1 marks : substantial progress towards correct solution
	(e)(i) $T_1 = 3$ $T_2 = 5$ $T_3 = 7$ $a = 3, \quad d = 2$ $T_n = 3 + 2(n - 1)$ $= 3 + 2n - 2$ $= 2n + 1$ $2n + 1 = 45$ $n = 22$ $\therefore$ 22 regular hexagons	2 marks : correct solution 1 marks : substantial progress towards correct solution
H5	**(ii)** Total perimeter of the 22 hexagons $= (6\times 3) + (6\times 5) + (6\times 7) + \ldots. + (6\times 45)$ $= 6(3 + 5 + 7 + \ldots + 45)$ $= 6\left(\frac{22}{2}(3 + 45)\right)$ $= 66\times 48$ $= 3\ 168\,\text{cm}$ Total length of the web $= 3168 + (6\times 45)$ $= 3\ 438$ cm	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution

Question No. 16 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 - applies appropriate techniques from the study of **calculus** to solve problems
H8 - uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
	(a) $\frac{dT}{dk}=\frac{t}{20}-k$ $T=\frac{t^2}{40}-kt+C$ when $t=0, T=100$ so, $100=\frac{0}{40}-0k+C$ $\therefore\ C=100$ and so $T=\frac{t^2}{40}-kt+100$ when $t=20,\ T=30$ and so $30=\frac{20^2}{40}-20k+100$ $20k=80$ $k=4$ $T=\frac{t^2}{40}-4t+100$	**<u>3 marks:</u>** correct solution **<u>2 marks:</u>** substantially correct solution **<u>1 mark:</u>** partial progress towards correct solution
	(b) x-intercepts are $-1, 3$ so rotated area is between -1 & 0 $V=\pi\int_{-1}^{0}y^2dx$ $=\pi\int_{-1}^{0}\left(4-(x-1)^2\right)dx$ $=\pi\left[4x-\frac{(x-1)^3}{3}\right]_{-1}^{0}$ $=\pi\left[\left(0-\frac{-1}{3}\right)-\left(-4-\frac{-8}{3}\right)\right]$ $=\frac{5\pi}{3}\ \text{u}^3$	**<u>3 marks:</u>** correct solution **<u>2 marks:</u>** substantially correct solution **<u>1 mark:</u>** partial progress towards correct solution

	(c) (i) $\frac{dV}{dt}=\frac{24t}{t^2+15}=12.\frac{2t}{t^2+15}$ $V=12\ln\left(t^2+15\right)+C$ $0=12\ln\left(0^2+15\right)+C \qquad \left(t=0\Rightarrow V=0\right)$ $C=-12\ln 15$ $V=12\ln\left(t^2+15\right)-12\ln 15$ $=12\ln\left(\frac{t^2+15}{15}\right)$	**3 marks:** correct solution **2 marks:** substantially correct solution **1 mark:** partial progress towards correct solution
	(c) (ii) $V=12\times 2=24\text{ m}^3$ $V=12\ln\left(\frac{t^2+15}{15}\right)$ $24=12\ln\left(\frac{t^2+15}{15}\right)$ $\ln\left(\frac{t^2+15}{15}\right)=2$ $\frac{t^2+15}{15}=e^2$ $t^2+15=15e^2$ $t^2=15e^2-15$ $=95\cdot 836\text{ hours}$ $t=\pm 9\cdot 7896\text{ hours}$ $=\ 9\text{ h }47\text{ min}$ ie tank will be full at 11:47pm Sunday	**3 marks:** correct solution **2 marks:** substantially correct solution **1 mark:** partial progress towards correct solution NB: this question was asking ***when*** the tank would be full, not how long it will take to fill.
	(c) (iii) at 6 pm, $t=4$ so $V=12\ln\left(\frac{4^2+15}{15}\right)=12\ln\frac{31}{15}$ $\frac{dV}{dt}=\frac{t^2}{k}$ $V=\frac{t^3}{3k}+C$ $12\ln\frac{31}{15}=\frac{0^3}{3k}+C \qquad \text{(start emptying at } t=0)$ $V=\frac{t^3}{3k}+12\ln\frac{31}{15}$ $0=\frac{2^3}{3k}+12\ln\frac{31}{15} \qquad \text{(empty when } t=2)$ $\frac{8}{3k}=-12\ln\frac{31}{15}$ $3k=-\frac{8}{12\ln\frac{31}{15}}$ $k=-0\cdot 3061$	**3 marks:** correct solution **2 marks:** substantially correct solution **1 mark:** partial progress towards correct solution